

# Casimir scaling hypothesis on the nonperturbative force in QCD vs. dual superconducting scenario of confinement\*

Y. Koma<sup>a</sup>, M. Koma<sup>a</sup>, and H. Toki<sup>b</sup>

<sup>a</sup> Max-Planck-Institut für Physik, Föhringer Ring 6, D-80805 München, Germany

<sup>b</sup> Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan

We discuss the Casimir scaling hypothesis on the nonperturbative force in terms of the dual superconducting picture of the QCD vacuum by calculating the string tensions of flux tubes associated with static charges in various  $SU(3)$  representations in the dual Ginzburg-Landau (DGL) theory.

## 1. Introduction

From the  $SU(2)$  and  $SU(3)$  lattice QCD studies of the static potential between color charges in various dimensional representations, there arises the Casimir scaling hypothesis for the intermediate distance force [1–3]. This hypothesis tells that the ratio of forces associated with various dimensional representations of color charges, for a given color group, is determined by that of eigenvalues of the quadratic Casimir operator for each representation.

It is expected that such a hypothesis is realized for the short distance force as described by one-gluon exchange, since the coupling is proportional to the quadratic Casimir operator. However, it is hard to imagine that this property is kept until intermediate distance where the nonperturbative effects set in. If the behavior of the ratio is governed exclusively by the group theoretical factor it should be manifest in arbitrary  $SU(N)$  gauge theory. Recent studies of  $k$ -strings in  $SU(4)$  and  $SU(6)$  lattice gauge theories, however, do not support Casimir scaling [4,5]. Therefore, it seems natural to consider that there are nonperturbative dynamics, rather than Casimir factor, which has inspired the Casimir scaling hypothesis for the intermediate distance force in early lattice investigations.

In this paper, we show that the dual superconducting picture of the nonperturbative QCD vacuum, as practically described by the dual Ginzburg-Landau (DGL) theory, provides us an understandable idea to explain the mechanism hidden behind the lattice data. In the dual superconducting vacuum, the color-electric flux is squeezed almost one dimensional tube, called the flux tube, due to the dual Meissner effect. For the quark and the antiquark system, then the flux tube is formed between the sources, leading to the linear potential, where its slope is identified as the string tension.

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## 2. Flux-tube solution in the DGL theory

We examine the SU(3) case and calculate the string tensions of flux tubes in the DGL theory associated with static charges in various SU(3) representations systematically [6,7], starting from the manifestly Weyl symmetric form [8,9]

$$\mathcal{L}_{\text{DGL}} = \sum_{i=1}^3 \left[ -\frac{1}{6g^2} {}^*F_{i\mu\nu}^2 + |(\partial_\mu + iB_{i\mu})\chi_i|^2 - \lambda (|\chi_i|^2 - v^2)^2 \right], \quad (1)$$

$${}^*F_{i\mu\nu} = \partial_\mu B_{i\nu} - \partial_\nu B_{i\mu} + 2\pi \sum_{j=1}^3 m_{ij} \Sigma_{j\mu\nu}^{(e)}. \quad (2)$$

Here  $B_{i\mu}$  and  $\chi_i$  ( $i = 1, 2, 3$ ) denote the dual gauge field and the complex scalar monopole field, respectively. The dual gauge fields within the Weyl symmetric expression have a constraint  $\sum_{i=1}^3 B_{i\mu} = 0$ . The  $\Sigma_{j\mu\nu}^{(e)}$  ( $j = 1, 2, 3$ ) in the dual field strength tensor is the color-electric Dirac string and their boundaries define the quark current  $j_{j\mu}^{(e)}$  ( $j = 1, 2, 3$ ).

The color-electric charge of the quark is specified by the weight vector of the SU(3) algebra,  $\vec{w}_j$  ( $j = 1, 2, 3$ ), as  $\vec{Q}_j^{(e)} \equiv e\vec{w}_j$ , where  $\vec{w}_1 = (1/2, \sqrt{3}/6)$ ,  $\vec{w}_2 = (-1/2, \sqrt{3}/6)$ , and  $\vec{w}_3 = (0, -1/\sqrt{3})$ . On the other hand, the color-magnetic charges of the monopole fields  $\chi_i$  are expressed by the root vectors of the SU(3) algebra,  $\vec{\epsilon}_i$ , as  $\vec{Q}_i^{(m)} \equiv g\vec{\epsilon}_i$  ( $i = 1, 2, 3$ ), where  $\vec{\epsilon}_1 = (-1/2, \sqrt{3}/2)$ ,  $\vec{\epsilon}_2 = (-1/2, -\sqrt{3}/2)$ , and  $\vec{\epsilon}_3 = (1, 0)$ . These color-electric and color-magnetic charges satisfy the extended Dirac quantization condition  $\vec{Q}_i^{(m)} \cdot \vec{Q}_j^{(e)} = 2\pi m_{ij}$ , where  $m_{ij} = 2\vec{\epsilon}_i \cdot \vec{w}_j$  and  $eg = 4\pi$ . There are two mass scales in the DGL theory: the masses of the dual gauge boson  $m_B = \sqrt{3}gv$  and the the monopole field  $m_\chi = 2\sqrt{\lambda}v$ . In analogy to usual superconductors, their ratio,  $\kappa \equiv m_\chi/m_B$ , corresponds to the Ginzburg-Landau (GL) parameter, which describes the type of dual superconductivity of the vacuum.

The flux-tube solution is obtained by considering the cylindrical symmetric system with translational invariance along the  $z$  axis. The fields are described as functions of radius  $r$  and azimuthal angle  $\varphi$ . Thus, we write the modulus of the monopole field as  $\phi_i(r) = |\chi_i(r)|$ . The dual gauge field is parametrized as  $\mathbf{B}_i = [B_i^{\text{reg}}(r) + B_i^{\text{sing}}(r)]\mathbf{e}_\varphi$  where  $B_i^{\text{reg}}(r) = [\tilde{B}_i^{\text{reg}}(r)/r]$  and  $B_i^{\text{sing}}(r) = -n_i^{(m)}/r$  with  $n_i^{(m)} \equiv \sum_{j=1}^3 m_{ij}n_j^{(e)}$ . Here  $n_j^{(e)}$  is the winding number of  $j$ -type color-electric Dirac string  $\Sigma_{j\mu\nu}^{(e)}$ , which takes various integers depending on the representation of the SU(3) color group to which the charges belong. For a given representation  $D$  of the SU(3) group, denoted by the Dynkin index  $[p, q]$ , we have  $\{n_1^{(e)}, n_2^{(e)}, n_3^{(e)}\} = \{p, -q, 0\}$  [10].

The string tension of the flux tube is calculated as an energy per unit length in  $z$  direction:

$$\sigma_D = 2\pi \sum_{i=1}^3 \int_0^\infty r dr \left[ \frac{1}{3g^2} \left( \frac{1}{r} \frac{d\tilde{B}_i^{\text{reg}}}{dr} \right)^2 + \left( \frac{d\phi_i}{dr} \right)^2 + \left( \frac{\tilde{B}_i^{\text{reg}} - n_i^{(m)}}{r} \right)^2 \phi_i^2 + \lambda(\phi_i^2 - v^2)^2 \right]. \quad (3)$$

In the Bogomol'nyi limit,  $\kappa = m_\chi/m_B = 1$ , this expression is analytically evaluated as [6,7,11]

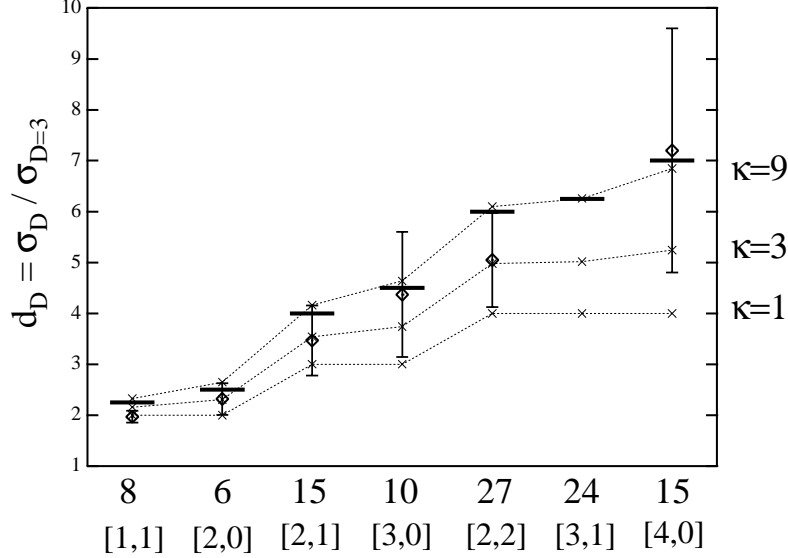


Figure 1. The ratios of the string tensions of flux tubes for various SU(3) representations,  $d_D = \sigma_D / \sigma_{\mathbf{3}}$  for the GL parameters  $\kappa = 1, 3$  and  $9$  (represented by crosses, each case connected by lines to guide the eye). The ratios of eigenvalues of the quadratic Casimir operators are shown as black bars. For comparison the lattice data of Ref. [2] are also plotted (diamonds with error bars). Boldface numbers and brackets  $[p, q]$  denote the dimension and the Dynkin indices of each representation  $D$ , respectively

In Fig. 1, we show the ratios of the string tensions of the flux tubes,  $d_D = \sigma_D / \sigma_{\mathbf{3}}$  for three values of the GL parameter,  $\kappa = 1, 3$ , and  $9$  (numerically obtained for  $\kappa \neq 1$ ). We also plot the ratios of the string tensions obtained by the lattice simulations of Ref. [2] and the ratios of eigenvalues of the quadratic Casimir operator,

$$C^{(2)}(D) = \frac{1}{3}(p^2 + pq + q^2) + (p + q). \quad (4)$$

We find that the DGL result in the type II dual superconducting vacuum near  $\kappa = 3$  agrees well with all lattice data obtained in Ref. [2], albeit with big errors. The mechanism of the  $\kappa$  dependence is understood as follows. In the Bogomol'nyi limit,  $\kappa = 1$ , the ratio between the string tensions of a higher and the fundamental representation satisfies the *flux counting rule*: the string tension  $\sigma_D$  is simply proportional to the number of the color-electric Dirac strings inside the flux tube, as seen from Eq. (??). With increasing  $\kappa$ , the interaction ranges of these fields get out of balance, and an excess of energy appears because of the interaction between fundamental flux tubes. This leads to systematic deviations from the counting rule. Note that the deviation of  $d_D$  from the counting rule grows toward higher representations  $D$ , since the number of fundamental flux which coexist in the flux tube of representation  $D$  increases as the sum  $p + q$  of Dynkin indices. On the other hand, we also find that the DGL result at  $\kappa = 9$ , for the deeply type II vacuum, uniformly reproduces *Casimir-like* ratios, through the deviations from the flux counting rule. In this analysis the higher dimensional flux tube in type II vacuum is assumed to be stable against splitting into fundamental ones. In principle, there must be

a certain minimal  $q\text{-}\bar{q}$  distance where such a effect is not negligible, depending on the GL parameter. However, in any case, the string tensions are saturated by the values at the Bogomol'nyi limit even if the splitting takes place.

### 3. Summary

We have studied the string tensions of flux tubes associated with static charges in various SU(3) representations in the DGL theory, based on a manifestly Weyl symmetric procedure. We have found that a GL-parameter near  $\kappa = 3$  reproduces the ratios of string tensions consistent with the lattice data [2]. The DGL theory accidentally shows Casimir-like scaling for a deeply type II vacuum with  $\kappa = 9$ . However, there is no direct relation to the eigenvalues of the Casimir operator. The mechanism of the systematic behavior of string tensions in the DGL theory can be understood as a result of the flux-tube dynamics. At present, it is not obvious that lattice data really contain such a dynamical effect. However, this example suggests that it is important to have more lattice results carefully interpreted without bias toward the Casimir scaling hypothesis.

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